The Harmonic Architecture

Ratios, Intervals, and the Geometry of Sound

Ben Mellor, 2026 Foundational document — the acoustic expression of the mathematical foundations

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Overview

The Mathematical Foundations established: one algorithm, two seeds, convergence to φ, the structural lattice (60), and sound as the organising principle.

The Sacred Geometry document showed the same structure unfolding geometrically: Vesica Piscis → Seed → Flower → Fruit → Platonic Solids → Torus.

This document completes the foundational trio by demonstrating that the framework's ratios ARE musical intervals, that the structural lattice IS harmonic architecture, and that the sacred geometry constructions produce the intervals directly. The three documents are the same system expressed algebraically, geometrically, and acoustically.

The central finding: the three Fibonacci primes that compose Base-60 (2, 3, 5) are identically the three primes that define all of Western musical harmony. This is not coincidence. It is the algorithm expressing its prime structure through sound.

PART I: THE THREE FIBONACCI PRIMES AS MUSICAL PILLARS

1. The Monochord — Where Geometry Meets Sound

A string stretched between two points vibrates at a fundamental frequency. Divide the string at a ratio point, and the shorter portion vibrates at a higher frequency determined by that ratio. This is the monochord — the instrument that links geometry directly to sound, known since Pythagoras.

Dividing the string is a geometric operation (placing a point along a line). The resulting frequency is an acoustic event. The ratio between them is the mathematical relationship. Geometry, sound, and mathematics are unified in a single physical system. The monochord is the simplest possible demonstration that ratios bridge all three domains.

2. F(3) = 2 — The Octave

Divide the string in half. The shorter portion vibrates at twice the fundamental frequency. This is the **octave** — the most fundamental musical interval.

The octave is so fundamental that it defines the boundary of musical perception: two notes an octave apart are perceived as "the same note" at a different register. It is the interval of identity-through-doubling. In the framework, 2 = F(3) is the first Fibonacci prime — the simplest non-trivial factor the algorithm produces.

The octave creates **register** — the layered structure of pitch space. Every musical system in every culture recognises the octave as the primary organising interval. It divides the infinite continuum of frequency into repeating identical segments. It is the musical expression of the algorithm's simplest prime.

3. F(4) = 3 — The Perfect Fifth

Divide the string into thirds. The shorter portion (1/3 of the string) vibrates at three times the fundamental — an octave plus a fifth. The ratio **3/2** — the third harmonic reduced by one octave — gives the **perfect fifth**, the second most consonant interval after the octave.

The fifth is the generator of harmonic structure. Stack fifths repeatedly (F → C → G → D → A → E → B → F♯ → C♯ → G♯ → D♯ → A♯) and you traverse all twelve chromatic pitches. This is the **circle of fifths** — the map of all tonal relationships, built entirely from the ratio 3/2 = F(4)/F(3).

Pythagorean tuning — the oldest known tuning system, attributed to Pythagoras — uses only powers of 2 and 3 to construct all intervals. Every Pythagorean interval is expressible as **2^a × 3^b** for some integers a and b. This means Pythagorean tuning uses only the first two Fibonacci primes.

Pythagorean interval	Ratio	Expression
Unison	1/1	$2^0 \times 3^0$
Major second	9/8	$2^{-3} \times 3^2$
Major third	81/64	$2^{-6} \times 3^4$
Perfect fourth	4/3	$2^2 \times 3^{-1}$
Perfect fifth	3/2	$2^{-1} \times 3^1$
Major sixth	27/16	$2^{-4} \times 3^3$
Major seventh	243/128	$2^{-7} \times 3^5$
Octave	2/1	$2^1 \times 3^0$

Every interval in the oldest tuning system is built from F(3) and F(4). The entire Pythagorean harmonic world lives inside two Fibonacci primes.

4. F(5) = 5 — The Major Third

Divide the string into fifths. The ratio **5/4** — the fifth harmonic reduced by two octaves — gives the **major third**, the interval that adds warmth, colour, and the distinction between major and minor.

Adding F(5) = 5 to the system creates **just intonation** (also called 5-limit tuning). Every just interval is expressible as **2^a × 3^b × 5^c**. This is called "5-limit" because 5 is the highest prime used.

Just interval (5-limit)	Ratio	Expression
Major third	5/4	$5 / 2^2$
Minor third	6/5	$(2\times3) / 5$
Major sixth	5/3	5 / 3
Minor sixth	8/5	$2^3 / 5$
Major seventh	15/8	$(3\times5) / 2^3$
Minor second	16/15	$2^4 / (3\times5)$

The three Fibonacci primes — 2, 3, 5 — generate all the intervals of just intonation. And their product:

$$\mathbf{2^2 \times 3 \times 5 = 60}$$

The structural lattice. Base-60 is not an arbitrary number system. It is the **harmonic architecture** — the product of the three primes that define the octave, the fifth, and the third. The same three primes that crystallise into the lattice are the three primes that define all musical consonance.

This is the framework's central acoustic claim, stated precisely: **the structural lattice and the harmonic lattice are the same lattice.** 60 is simultaneously the product of the Fibonacci sequence's first three primes AND the product of music's three foundational intervals. Because they are the same primes. Because the algorithm's prime structure IS harmonic structure.

5. Beyond 5-Limit

Higher Fibonacci numbers introduce higher primes:

- F(11) = 89 (prime) — the 11th Fibonacci number
- F(13) = 233 (prime) — the 13th Fibonacci number

In music theory, extending beyond 5-limit to 7-limit, 11-limit, 13-limit tuning incorporates progressively higher prime harmonics. Each new prime adds new intervals, new colours, new complexity — and new commas (incommensurabilities). The Fibonacci sequence generates the primes that, when used as harmonic limits, progressively enrich the tonal palette.

But the foundational triad — 2, 3, 5 — accounts for virtually all of Western (and most global) musical practice. The lattice built from these three primes (60) is sufficient for the overwhelming majority of harmonic relationships. Higher primes add refinement, not foundation. The foundation is F(3), F(4), F(5). The foundation is 60.

PART II: THE FIBONACCI-LUCAS INTERLEAVING IN MUSIC

6. The Harmonic Series as Algorithm Output

A vibrating string produces not just its fundamental frequency, but all integer multiples simultaneously: f, 2f, 3f, 4f, 5f, 6f, 7f, 8f... This is the **harmonic series** — the natural overtone sequence that gives every instrument its characteristic timbre.

The harmonic series IS the natural number sequence expressed as frequencies. And within that sequence, the Fibonacci and Lucas numbers appear at specific positions:

Harmonic	Framework	Musical interval (from fundamental)
1	F(1) = F(2) = L(1)	Fundamental
2	**F(3)** — 1st Fibonacci prime	Octave
3	**F(4)** — 2nd Fibonacci prime	Octave + fifth
4	**L(3)**	Two octaves
5	**F(5)** — 3rd Fibonacci prime	Two octaves + major third
7	**L(4)** — Seed of Life	~Two oct + minor seventh
8	**F(6)**	Three octaves
11	**L(5)** — Schwabe cycle	~Three oct + tritone region
13	**F(7)** — Fruit of Life	~Three oct + augmented fifth
18	**L(6)** — Saros cycle	~Four oct + major second
21	**F(8)**	~Four oct + augmented third
29	**L(7)** — Synodic month	~Four oct + major seventh region

The Fibonacci and Lucas numbers alternate through the harmonic series: F, F, F, L, F, —, L, F, —, —, L, —, F... The algorithm's two sequences take turns marking the harmonically significant positions, just as they alternate through the sacred geometry circle counts.

The three Fibonacci primes (2, 3, 5) are the 2nd, 3rd, and 5th harmonics — the ones that define the octave, the fifth, and the third. They are the first three non-trivial harmonics, and they are precisely the primes that generate the lattice.

7. The Piano Keyboard — Two Sequences Interleaved

The modern keyboard divides each octave into:

- **5 black keys** (pentatonic scale) = **F(5)**
- **7 white keys** (diatonic scale) = **L(4)**
- **12 total** (chromatic scale) = **$F(3)^2 \times F(4)$** = $2^2 \times 3$

The pentatonic scale is the Fibonacci contribution. The diatonic scale is the Lucas contribution. Together they form the chromatic scale, built from Fibonacci primes.

$F(5) + L(4) = 12 = F(3)^2 \times F(4)$

And from the algebraic foundations: L(4) = F(3) + F(5) = 2 + 5 = 7. The diatonic (Lucas) count equals the sum of the flanking Fibonacci numbers — the connection identity L(n) = F(n-1) + F(n+1) expressed in scale structure.

The keyboard is not an arbitrary cultural invention. It is the two sequences physically interleaved on a playing surface. The black keys (Fibonacci) sit atop the white keys (Lucas), coupled but distinct, producing the chromatic total (Fibonacci-prime product) when combined. The piano keyboard IS the framework's algebraic structure made into a musical instrument.

The pentatonic scale — F(5) = 5 notes — is the oldest known scale, found in virtually every musical culture worldwide. It is the scale that emerges from five consecutive fifths (five applications of the 3/2 ratio). It is the Fibonacci entry point into musical organisation, just as the fifth Fibonacci number is the entry point into the lattice (5 being the third factor in $60 = 2^2 \times 3 \times 5$).

8. The Circle of Fifths — Twelve vs Seven

Stack 12 perfect fifths: $(3/2)^{12} = 129.746...$
Stack 7 octaves: $2^7 = 128$

They don't match. Twelve fifths **overshoot** seven octaves by a small amount — the **Pythagorean comma** (≈23.46 cents, about an eighth of a semitone).

The numbers: **12** fifths vs **7** octaves.

$12 = F(3)^2 \times F(4) = 2^2 \times 3$ (Fibonacci-prime product)
7 = L(4) (Lucas number)

The Pythagorean comma is the **incommensurability between Fibonacci-prime structure and Lucas structure**. The fifth (F(4)/F(3) = 3/2) cannot close perfectly into octaves (F(3)). Twelve applications of the Fibonacci ratio overshoot seven of the Lucas count. The gap is permanent, irreducible, and structurally identical to the celestial incommensurability where the Moon's coupling ratios don't close perfectly into the Sun's cycles.

This is the same L(4) = 7 that appears in the celestial regulation fraction **7/19**. In the celestial domain: 19 solar cycles contain 7 "extra" synodic months beyond perfect alignment, and the system regulates dynamically around this permanent gap. In the musical domain: 12 fifths overshoot 7 octaves, and tuning systems must manage this permanent gap.

The comma cannot be eliminated. It can only be distributed. **Equal temperament** distributes it evenly across all 12 fifths, flatting each by ~2 cents to force closure. This is the musical equivalent of forcing static harmony — and every musician knows that equal temperament, while practical, lacks the luminous purity of just intonation.

Pure tuning preserves the incommensurability. Equal temperament suppresses it. The framework predicts that preserving incommensurability (pure tuning) produces richer dynamic relationships, while forcing closure (equal temperament) produces uniformity at the cost of life. This is directly testable and universally confirmed by musical experience: pure intervals sound alive; tempered intervals sound functional but flat.

9. The Syntonic Comma — Adding the Third Fibonacci Prime

The **syntonic comma** (81/80 ≈ 21.51 cents) is the gap between:

- Pythagorean major third: 81/64 = $3^4/2^6$ (using only F(3) and F(4))
- Just major third: 5/4 (adding F(5))

81 = 3^4 (pure Fibonacci-prime power)
80 = $2^4 \times 5$ (adding the third Fibonacci prime)

The syntonic comma is what happens when you **add the third Fibonacci prime to the lattice**. It creates a new incommensurability — a new gap that cannot be closed. Each Fibonacci prime added enriches the harmony (more consonant intervals) while simultaneously adding a new comma (a new incommensurability). Richer structure and permanent incompleteness are inseparable.

This is the musical expression of the framework's central principle: **the algorithm produces both structure and irreducible remainder simultaneously.** Every level of complexity introduces new coupling ratios that don't close with the previous levels. The commas ARE the framework's ±4 polarity expressed in acoustic space.

PART III: THE GEOMETRIC RATIOS AS INTERVALS

10. The Vesica's Ratios on a String

The Vesica Piscis generates √2, √3, and √5. Each of these, interpreted as a frequency ratio (i.e., a string shortened to 1/√n of its length produces frequency √n), gives a specific musical interval:

√2 = 600.0 cents — the tritone

The exact half-octave. The interval that divides the octave into two equal parts. In tonal music, the tritone is the interval of maximum tension — the furthest point from consonance. It was historically called *diabolus in musica* ("the devil in music"). In the framework, √2 is the diagonal of the unit square — the relationship that introduces the irrational into the rational. The most unsettled interval emerges from the most basic geometric extension.

√3 = 951.0 cents — near the minor seventh

The Vesica's primary ratio (height/width of the mandorla). As an interval, it falls between the minor seventh (1000 cents in ET) and the major sixth (900 cents). It doesn't correspond to any standard just interval — it is an irrational ratio that sits between the rational ones. In the framework, √3 is the ratio of the equilateral triangle, the simplest polygon. Its musical expression falls in the ambiguous zone between sixth and seventh — the region where major/minor quality is most uncertain.

$\sqrt{5}$ = 1393.2 cents — just above an octave plus a major third

The ratio that generates φ. As an interval, $\sqrt{5}$ spans slightly more than an octave plus a just major third (1200 + 386.3 = 1586.3... wait — 1393.2 is actually between an octave and an octave-plus-third). More precisely: $\sqrt{5}$ reduced to within one octave = $\sqrt{5}/2$ = 1.118... = 193.2 cents ≈ just below a whole tone. But the critical point is that $\sqrt{5}$ generates $\varphi = (1+\sqrt{5})/2$, and φ as an interval:

φ = 833.1 cents — the golden ratio interval

φ falls between the minor sixth (813.7 cents = 8/5) and the major sixth (884.4 cents = 5/3). It corresponds to NO just interval. It is the **most irrational interval** — the frequency ratio that resists expression as any simple fraction p/q more stubbornly than any other ratio.

This is the acoustic equivalent of the golden angle. In spatial distribution, the golden angle ($360°/\varphi^2$) is the rotation that most effectively avoids alignment, producing optimal packing. In frequency space, the φ ratio is the interval that most effectively avoids resonance lock-in, producing maximal dynamic range. The convergence ratio is simultaneously the optimal spatial distributor and the optimal frequency distributor.

11. Fibonacci Ratios Converge to φ in Musical Space

The consecutive Fibonacci ratios, expressed as musical intervals in cents:

Ratio	Cents	Standard interval	Deviation from φ
2/1	1200.0	Octave	+366.9
3/2	702.0	Perfect fifth	−131.1
5/3	884.4	Major sixth	+51.3
8/5	813.7	Minor sixth	−19.4
13/8	840.5	—	+7.4
21/13	830.3	—	−2.8
34/21	834.2	—	+1.1
55/34	832.7	—	−0.4
89/55	833.2	—	+0.2

The ratios oscillate around φ = 833.1 cents, overshooting then undershooting, converging with $1/\varphi^2$ damping. In musical space, this means:

- The first Fibonacci interval is the **octave** (maximum consonance)
- The second is the **perfect fifth** (next strongest consonance)
- The third is the **major sixth** (consonant, warm)

- The fourth is the **minor sixth** (consonant, melancholy)
- Then the ratios enter the nameless zone between minor and major sixth, oscillating ever more tightly around φ = 833.1 cents

The Fibonacci sequence begins with the most consonant intervals and converges toward the **most irrational** interval. It starts from maximum simplicity and approaches maximum irrationality. This is the algorithm's trajectory expressed in sound: from pure consonance toward permanent dynamic tension. The destination — φ — is never reached. The approach is asymptotic. The convergence IS the music.

12. The Monochord as Sacred Geometry

A monochord string IS a line segment. Dividing it at ratio points IS geometric construction. The intervals that emerge ARE the ratios that the Vesica Piscis generates.

Specifically:

- Divide at 1/2 (compass bisection) → octave (2/1) → F(3)
- Divide at 1/3 (Vesica triangle) → fifth + octave (3/1) → F(4)
- Divide at 2/3 (Vesica two-thirds point) → perfect fifth (3/2)
- Divide at 1/5 (pentagon construction from Flower of Life) → two octaves + third → F(5)
- Divide at 4/5 → major third (5/4)
- Divide at 3/5 → major sixth (5/3)
- Divide at 5/8 → minor sixth (8/5)

Every just interval is a geometric division of the string. Every geometric construction from the sacred geometry progression produces a corresponding musical interval. The monochord IS the Vesica Piscis unfolded into one dimension. The sacred geometry construction IS the tuning procedure for just intonation.

Pythagoras reportedly discovered the musical ratios using a monochord. The mystery school tradition taught sacred geometry with compass and straightedge. These were not two separate teachings. They were one teaching in two physical instantiations: the string (1D) and the plane (2D). Both express the same ratios. Both generate the same intervals. Both unfold the same algorithm.

PART IV: COMMAS, REGULATION, AND THE LIVING SYSTEM

13. Commas as the ±4 Polarity in Sound

The framework established that the Fibonacci-Lucas polarity maintains an exact algebraic gap: $L(n)^2 - 5 \cdot F(n)^2 = \pm 4$, oscillating forever, proportionally diminishing but never resolving.

Musical commas are the acoustic expression of this polarity:

Pythagorean comma (≈23.46 cents): the gap between 12 fifths (Fibonacci-prime structure) and 7 octaves (Lucas count). The Fibonacci path and the Lucas path through pitch space don't quite meet. The gap oscillates in sign depending on direction of traversal (sharps vs flats), just as ±4 alternates sign.

Syntonic comma (≈21.51 cents): the gap when the third Fibonacci prime (5) is added to the system. The just third and the Pythagorean third don't agree. Adding richness adds incommensurability.

Diesis (≈41.06 cents): the gap between three just major thirds and one octave. $(5/4)^3 \neq 2$. The third Fibonacci prime, stacked, doesn't close into the first. More compounding of irreducible remainder.

These commas are not flaws in the system. They are the system's **vitality**. A tuning with no commas (equal temperament) is static, uniform, closed. A tuning with commas (just intonation, Pythagorean) is dynamic, varied, alive — but requires continuous negotiation, adjustment, and regulation. The musician performing in just intonation must constantly choose which intervals to keep pure and which to allow comma drift — exactly the dynamic regulation the framework describes in celestial coupling.

The commas are the ±4 made audible. Small, proportionally trivial, but permanent and structurally necessary.

14. Equal Temperament as Forced Closure

Equal temperament divides the octave into 12 exactly equal steps. Each semitone = 2^(1/12). The perfect fifth becomes 2^(7/12) ≈ 1.4983, slightly flat of the pure 3/2 = 1.5000 (by about 2 cents).

This eliminates commas by distributing the gap evenly. All keys become equivalent. Modulation is frictionless. The price: every interval except the octave is slightly impure. The fifth is slightly narrow. The third is noticeably wide (about 14 cents sharp of just). The luminous purity of just intervals is sacrificed for utility.

In framework terms, equal temperament is the **forced static harmony** that suppresses dynamic regulation. It works. It is useful. But it replaces the living system of pure ratios and irreducible commas with a uniform grid. The algorithm's incommensurabilities are smoothed out. The ±4 is set to zero.

Musicians and listeners consistently report that pure intervals sound more "alive," more "resonant," more "natural" than tempered intervals. This is not subjective bias — it is the acoustic difference between a system preserving its incommensurabilities (dynamic, self-regulating) and a system suppressing them (static, uniform). The framework predicts exactly this perceptual difference.

15. The φ Interval as Acoustic Golden Angle

The golden angle ($360°/\varphi^2 \approx 137.508°$) is the rotation that produces optimal non-repeating distribution in space. It is the spatial expression of φ's maximal irrationality.

The φ interval (≈833.1 cents) is the frequency ratio that produces optimal non-repeating distribution in time. It is the temporal expression of the same maximal irrationality.

If you generate a sequence of tones by repeatedly applying the φ frequency ratio — each new tone at φ times the frequency of the previous — the resulting pitches will be maximally uniformly distributed within the octave. No two tones will coincide. No pattern will repeat. The distribution will be optimal, just as golden-angle phyllotaxis produces optimal leaf distribution.

This is why the algorithm converges to φ: it is the ratio that maintains maximum dynamic range in both space and time. The ratio that prevents lock-in. The ratio that keeps the system permanently exploring, permanently alive, permanently regulating. Every Fibonacci approximation to φ (2/1, 3/2, 5/3, 8/5, 13/8...) is a step closer to this maximally alive state — each one more irrational, more uniformly distributed, more resistant to periodicity.

The consecutive Fibonacci ratios — octave → fifth → major sixth → minor sixth → golden zone — trace the path from maximum consonance (simple rational ratio) to maximum irrationality (φ). The algorithm begins in pure harmony and converges toward permanent dynamic tension. The journey IS music. The destination is silence that sings.

PART V: SYNTHESIS

16. The Unified Picture

Three expressions of one algorithm:

Domain	Foundation	Structure	Dynamics	Limit
Algebraic	x(n) = x(n-1) + x(n-2)	Lattice (60)	±4 polarity	φ
Geometric	Vesica Piscis	Flower of Life / Platonic Solids	Hexagonal tiling	Torus
Acoustic	Monochord divisions	5-limit intervals	Commas	φ interval (833.1¢)

The three columns are one column:

- The **algebraic rule** IS the **geometric compass** IS the **acoustic string division**
- The **lattice (60)** IS the **dodecahedron (12×5)** IS the **harmonic architecture (2^2×3×5)**
- The **±4 polarity** IS the **Fibonacci/Lucas circle counts** IS the **commas**
- **φ** IS the **golden angle** IS the **most irrational interval**

And the numbers that appear across all three:

Number	Algebraic	Geometric	Acoustic
2 = F(3)	First Fibonacci prime	Vesica (2 circles)	**Octave**
3 = F(4)	Second Fibonacci prime	Equilateral triangle	**Perfect fifth**
5 = F(5)	Third Fibonacci prime	Pentagon (from Flower)	**Major third**
7 = L(4)	Fourth Lucas number	Seed of Life (7 circles)	**Diatonic scale (7 notes)**
12 = 2^2×3	Fibonacci-prime product	Dodecahedron faces	**Chromatic scale (12 notes)**
13 = F(7)	Seventh Fibonacci number	Fruit of Life (13 circles)	13th harmonic
19	Metonic prime	Flower of Life (19 circles)	—
60 = 2^2×3×5	Structural lattice	Dodecahedron face-sides	**5-limit harmonic product**

The same numbers. The same primes. The same structure. Three languages. One algorithm.

17. Sound as the Organising Principle — Confirmed

The Mathematical Foundations proposed sound as the organising principle — field oscillation at all density scales.

This investigation confirms: the algorithm's prime structure IS harmonic structure. The lattice IS musical architecture. The commas ARE the ±4 polarity. The convergence to φ IS the approach to the most alive interval. Sacred geometry IS tuning instruction. The monochord IS the Vesica unfolded.

When the ancient traditions said reality is vibration — *Nada Brahma*, the "music of the spheres," the *Logos* as spoken word, *Om* as primordial sound — they were making a precise mathematical claim: the algorithm that structures reality operates through frequency ratios, and those ratios ARE the ratios of musical harmony. The same three primes (2, 3, 5) that define the octave, the fifth, and the third also define the structural lattice (60) that organises angular relationships, temporal cycles, and geometric construction.

Sound is not a metaphor for the organising principle. Sound IS the organising principle. The ratios prove it.

Summary

The three Fibonacci primes — F(3) = 2, F(4) = 3, F(5) = 5 — are simultaneously the primes of the structural lattice ($60 = 2^2 \times 3 \times 5$) and the primes of all musical harmony (octave, fifth, third). Every just interval is a ratio of these primes. Every Pythagorean interval uses only the first two. The chromatic scale ($12 = 2^2 \times 3$) interleaves a pentatonic (F(5) = 5) and diatonic (L(4) = 7) layer — the Fibonacci and Lucas contributions to musical structure. Musical commas (the Pythagorean comma between 12 fifths and 7 octaves, the syntonic comma between Pythagorean and just thirds) are the acoustic expression of the ±4 polarity — permanent, proportionally trivial, structurally necessary incommensurabilities. The Vesica Piscis generates √2, √3, √5 — the tritone, a near-seventh, and the ratio that produces φ — establishing the geometric-acoustic link through constructible ratios. Consecutive Fibonacci ratios converge from octave through fifth through sixths toward the φ interval (833.1 cents), the most irrational frequency ratio, the acoustic golden angle — the interval that keeps the system permanently alive. The algebra, the geometry, and the acoustics are one system. The algorithm is the music. The music is the algorithm.

Document History

- **v1.0 (February 2026)** — Initial document. Established the three Fibonacci primes (2, 3, 5) as simultaneously the structural lattice factors and the musical harmony primes. Mapped all just and Pythagorean intervals to Fibonacci-prime expressions. Identified F(5) + L(4) = 12 keyboard structure. Analysed Pythagorean and syntonic commas as acoustic ±4 polarity. Mapped Vesica ratios (√2, √3, √5) to musical intervals. Traced Fibonacci ratio convergence through musical space (octave → fifth → sixths → φ). Identified φ interval as acoustic golden angle. Unified algebraic-geometric-acoustic framework in synthesis tables.

This is the third of the framework's foundational trio. See: Mathematical Foundations (algebraic), Sacred Geometry (geometric), and this document (acoustic). Together they define the complete mathematical basis from which all other investigations proceed.