The Thomson Bridge

How Energy Minimisation on a Sphere Generates the Golden Ratio Across Every Scale of Matter

Ben Mellor, 2026 — Development Document for the Toroidal Consciousness-EM Field Framework

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The Argument

The framework identifies two algorithms — The Weaving (Fibonacci: 1, 1, 2, 3, 5, 8, 13...) and The Loom (Lucas: 2, 1, 3, 4, 7, 11, 18...) — as the structural engines of physical reality. The Galactic Signature demonstrates that these algorithms govern orbital resonance ratios across independent star systems. The Chemical Foundations document shows they appear in atomic spectral lines and molecular bonding geometries.

But a persistent question remains: **why** do these particular numbers keep appearing? What is the physical mechanism that selects Fibonacci and Lucas over all other possibilities?

The Thomson problem provides the answer.

When you take any number of identical charges and confine them to a sphere, then ask nature to find the lowest-energy arrangement, the solutions spontaneously generate pentagonal symmetry, icosahedral geometry, and the golden ratio. Not because anyone designed them to. Because energy minimisation on a closed surface **mathematically requires** phi-based geometry.

This is the bridge. The Thomson problem connects the abstract mathematics of Fibonacci/Lucas sequences to the concrete physics of electrostatic energy minimisation — and it does so at every scale, from individual atoms to virus capsids to crystal lattices to the large-scale structure of the cosmos.

PART I: THE THOMSON PROBLEM — DEFINITION AND SOLUTIONS

1. What the Problem Is

In 1904, J.J. Thomson — the man who discovered the electron — posed a deceptively simple question:

Given N identical point charges confined to the surface of a unit sphere, what arrangement minimises the total electrostatic potential energy?

The energy to be minimised is:

> $E = \Sigma(i<j)\ 1/|\mathbf{r}_i - \mathbf{r}_j|$

where $\mathbf{r}_i$ is the position of the i-th charge. Every charge repels every other charge. The sphere constrains them. Nature finds the configuration where the total repulsion is as low as possible.

This is an NP-hard optimisation problem — there is no general analytical solution for arbitrary N. But for certain values of N, the solutions are known exactly. And they are remarkable.

2. The Exact Solutions and Their Geometries

N	Minimum energy configuration	Geometry	Phi content
2	Antipodal points	Line segment	—
3	Equilateral triangle on great circle	Triangle	—
4	Regular tetrahedron	Platonic solid	—
5	Triangular dipyramid	Johnson solid	Pentagonal local minimum exists
6	Regular octahedron	Platonic solid	—
8	Twisted rectangular parallelepiped	**Not** a cube	—
12	**Regular icosahedron**	**Platonic solid**	**φ throughout**
20	**Not** a dodecahedron	Irregular	—
32	Icosahedral symmetry	Buckminster structure	**φ throughout**
72	Icosahedral symmetry	Caspar-Klug lattice	**φ throughout**
122	Icosahedral symmetry	Extended CK lattice	**φ throughout**
132	Icosahedral symmetry	Extended CK lattice	**φ throughout**
192	Icosahedral symmetry	Extended CK lattice	**φ throughout**

The critical observation: The cube (N=8) and dodecahedron (N=20) are **not** energy minima. Nature rejects them. But the icosahedron (N=12) **is** the exact global minimum. Nature selects it.

The icosahedron is the Platonic solid most saturated with the golden ratio. Its vertex coordinates on a unit sphere can be written as permutations of $(0, \pm 1, \pm \varphi)$. The ratio of edge length to circumradius is $1/\sin(2\pi/5)$ — a function of the pentagon angle, which is defined by φ. The dihedral angle is $2 \cdot \arcsin(\varphi/2)$. Every metric of the icosahedron encodes the golden ratio.

Nature does not choose the icosahedron because it is pretty. It chooses it because it is the lowest energy state. And that state is defined by φ.

3. The Magic Numbers

Numerical studies of the Thomson problem for N up to 4,352 (Wales & collaborators, using basin-hopping algorithms) have revealed a set of "magic numbers" — values of N where the energy minimum is anomalously low relative to the smooth trend curve. These magic numbers correspond to configurations with icosahedral symmetry:

N = 12, 32, 72, 122, 132, 192, 212, 272, 282, 312, 372...

These follow the Caspar-Klug formula: **$N = 10(m^2 + n^2 + mn) + 2$**, where m and n are positive integers. The formula itself encodes the triangulation number $T = m^2 + n^2 + mn$, which governs the geometry of every icosahedral structure in nature — from virus capsids to fullerene cages.

The vibrational analysis confirms this (2005 study): the maximum vibrational frequency ω_{max} scales as $N^{(3/4)}$, but shows sharp discontinuities at the magic numbers, reflecting both strong degeneracy of one-particle energies and icosahedral structure.

The Thomson problem does not merely permit icosahedral symmetry. It preferentially selects it at specific magic numbers — the same numbers that govern the architecture of viruses and carbon nanostructures.

4. Large N: The Golden Angle and the 12 Pentagonal Defects

For large N (hundreds to thousands of charges), the solutions reveal a striking pattern:

The charges arrange themselves into a **triangular lattice** (the most efficient packing on a flat surface) but with exactly **12 pentagonal defects** — points where the local coordination drops from 6 to 5 neighbours.

This is not a design choice. It is a **topological requirement**. Euler's theorem for convex polyhedra states that $V - E + F = 2$, and this constrains any triangulated sphere to have exactly 12 vertices with 5-fold (pentagonal) coordination, regardless of how many vertices there are in total.

Twelve pentagons. Always twelve. The same number as the vertices of an icosahedron.

For the arrangement of charges within these lattices, the most efficient spacing uses the **golden angle**: $360°/\varphi^2 \approx 137.508°$. This is the same angle that governs phyllotaxis in plants — the arrangement of leaves, seeds, and petals that maximises exposure to light and rain. The golden angle produces the most uniform possible distribution on a sphere because φ is the "most irrational" number — its continued fraction convergents approach it more slowly than for any other irrational, meaning golden-angle spacing avoids the clustering that any rational or less-irrational angle would produce.

NASA applied this directly: the Starshine-3 satellite used golden-angle spacing to arrange 1,500 mirrors on its spherical surface for uniform light reflection.

5. The Continued Fraction Connection

A 2023 study in *Scientific Reports* (Moscato et al.) demonstrated that the Thomson problem's energy minima can be approximated with extraordinary precision using continued fractions — and that the golden ratio's unique continued fraction [1; 1, 1, 1, ...] plays a fundamental role.

For N = 4,352 charges, their continued-fraction approximation achieved a mean squared error of just 5.55×10^{-8} for the normalised energy. The residuals (deviations from the smooth approximation) correlate with number-theoretic properties — specifically with values of N where $N^2 + 12$ is prime.

The number 12 appears again. It is the topological charge of the sphere — the number of pentagonal defects required by Euler's theorem — and it is $L(1) \times L(4) = 2 \times (2^2 + (-1)^2 \cdot 2) = ...$ no. More simply: $12 = F(3)^2 \times F(4) = 4 \times 3$. The structural number of the Loom.

PART II: SCALE INVARIANCE — THE SAME GEOMETRY AT EVERY LEVEL

6. Atomic Scale: Electron Shells and Quantum Dots

The Thomson problem was originally conceived as a model for atomic structure (Thomson's "plum pudding" model). While the plum pudding model was superseded, the mathematical solutions remain directly relevant:

Electron shell magic numbers: In the jellium model for metallic clusters, electrons form shells that close at magic numbers (2, 8, 18, 20, 34, 40, 58, 92...) corresponding to especially stable Thomson-like configurations. Photoabsorption experiments on sodium and potassium clusters confirm these shell closures.

Quantum dots: Electrons confined in semiconductor quantum dots — "artificial atoms" — exhibit shell-filling patterns that mirror Thomson solutions. The energy level occupancies and ionisation thresholds correlate with discontinuities in the Thomson potential energy increments $\Delta U^{+}(N)$ for $N \leq 100$, reflecting the same icosahedral preference seen in the classical problem.

Small metallic clusters (N < 1,000 atoms): These exhibit icosahedral symmetry that aligns with Thomson minima — confirmed experimentally through mass spectrometry and photoelectron spectroscopy.

Charged helium nanodroplets: A 2024 He-DFT study calculated minimum droplet radii for hosting Z ions, showing that stable droplets form shells with magic numbers closely resembling Thomson equilibria — particularly N = 12 (icosahedral). The results are insensitive to ion species, confirming the universality of the geometric principle.

Framework interpretation: At the atomic scale, charges confined to spherical geometries spontaneously adopt icosahedral (φ-based) configurations because these minimise energy. The Weaving is not imposed — it emerges from electromagnetic optimisation.

7. Molecular Scale: Virus Capsids

Virus capsids are the most spectacular demonstration of Thomson geometry in biology. Hundreds of virus families construct their protein shells using icosahedral symmetry — the same geometry that solves the Thomson problem for N = 12.

The Caspar-Klug architecture:

Caspar and Klug (1962) showed that icosahedral virus capsids can be classified by a triangulation number $T = h^2 + k^2 + hk$ (h, k non-negative integers), giving the series:

> T = 1, 3, 4, 7, 12, 13, 16, 19, 21, 25, 27, 28...

Each capsid contains exactly **12 pentamers** (clusters of 5 protein subunits) plus **10(T−1) hexamers** (clusters of 6). The total number of protein subunits is 60T.

The 12 pentamers sit at the vertices of an icosahedron. The hexamers fill the triangular faces. This is precisely the Thomson solution for large N: a triangular lattice with 12 pentagonal defects, mapped onto a sphere.

Specific examples:

Virus	T-number	Total subunits	Icosahedral?
Satellite Tobacco Mosaic Virus	T = 1	60	Yes
L-A Virus	T = 2	120	Yes
Dengue Virus	T = 3	180	Yes
Sindbis Virus	T = 4	240	Yes
Bovine Papilloma Virus	T = 6	360	Yes
Adenovirus	T = 25	1,500	Yes
Mimivirus	T ≈ 1,000	~60,000	Yes

The 72-pentamer icosahedral capsids of the papovavirus family (which includes the oncogenic polyomavirus and HPV) use pentamers at all positions — both pentavalent and hexavalent. Their surface lattice was shown to correspond exactly to Fibonacci pentilings (Kiselev & Klug), where the characteristic length scale is τ^2 times the pentagon edge length ($\tau = \varphi$ = golden ratio).

The quasicrystalline connection: Twarock and colleagues (2011, 2019) demonstrated that viral capsids can be modelled as finite subsets of icosahedral 3D quasicrystals — the same aperiodic structures discovered by Schechtman in 1984 (Nobel Prize 2011). The capsid protein positions correspond to projections from a 6-dimensional hypercubic lattice, and structural transitions during viral maturation follow paths analogous to Bain deformations in crystallography, but in 6D icosahedral space.

Framework interpretation: Virus capsids are Thomson solutions in protein. The electromagnetic interactions between charged amino acid residues on capsid protein surfaces drive self-assembly toward the same icosahedral (φ-based) minima that pure point charges find on a sphere. The Weaving algorithm operates through protein folding and electrostatic self-assembly.

8. Molecular Scale: Fullerenes and Carbon Nanostructures

C_{60} — Buckminsterfullerene — is a truncated icosahedron: 60 carbon atoms arranged at the vertices of a shape with 12 pentagonal and 20 hexagonal faces. It is the molecular embodiment of the Thomson solution.

The fullerene family follows the same rules as virus capsids: 12 pentagonal rings (required by Euler's theorem) plus varying numbers of hexagonal rings. The isolated pentagon rule (IPR) — which states that the most stable fullerenes have no adjacent pentagons — produces structures where the 12 pentagons are maximally separated, exactly as Thomson charges would be.

Carbon onions (nested fullerene shells) exhibit the same icosahedral symmetry at multiple radial levels — a fractal-like nesting of φ-based geometry, layer within layer.

Framework interpretation: Carbon, the element most central to biological chemistry, spontaneously adopts icosahedral geometry when forming closed structures. The 12 pentagonal defects in every fullerene are the

Euler-mandated pentagonal signature of spherical topology — the same signature that appears in Thomson solutions, virus capsids, and the large-scale geometry of the cosmos.

9. Macroscopic Scale: Quasicrystals

In 1984, Daniel Schechtman observed electron diffraction patterns from rapidly cooled Al-Mn alloys that displayed sharp Bragg peaks with icosahedral symmetry — five-fold rotational symmetry, which is forbidden in periodic crystals. This was the discovery of quasicrystals, which earned Schechtman the 2011 Nobel Prize in Chemistry.

Key properties of icosahedral quasicrystals (IQCs):

- They have icosahedral symmetry at all scales — unlike periodic crystals, a shifted copy never exactly matches the original
- Their atomic positions can be described as projections from a 6-dimensional periodic lattice onto 3D space
- The projection involves the golden ratio: the "acceptance domain" in the perpendicular space has dodecahedral symmetry, and the scaling between physical and perpendicular space is governed by $\tau = \varphi$
- Their diffraction patterns can be indexed using Fibonacci pentilings, where the characteristic length scale is τ^2 times the fundamental tile edge

The Penrose tiling connection: The 2D analogue of a quasicrystal is a Penrose tiling — an aperiodic tiling of the plane using two tile shapes (kites and darts, or fat and thin rhombi) whose areas are in the ratio φ:1. The ratio of kites to darts (or fat to thin rhombi) in an infinite Penrose tiling converges to φ. Every measurement of a Penrose tiling encodes the golden ratio.

Natural quasicrystals: In 2009, Luca Bindi discovered the first natural quasicrystal — icosahedrite ($Al_{63}Cu_{24}Fe_{13}$) — in a meteorite from the Khatyrka region of Russia. The meteorite dates to the formation of the solar system, approximately 4.5 billion years ago. Quasicrystalline order with icosahedral symmetry has been present in nature since the birth of the solar system.

Recent developments: A 2020 study showed that IQCs can self-assemble from a single-component system of particles interacting via a relatively short-range isotropic pair potential — no complicated multi-body interactions required. Local geometric frustration alone is sufficient to stabilise icosahedral quasicrystalline order. This demonstrates that φ-based long-range order can emerge from the simplest possible interactions.

Framework interpretation: Quasicrystals are the macroscopic expression of icosahedral geometry — the same geometry that solves the Thomson problem and structures virus capsids. The golden ratio is not merely present in quasicrystals; it is the defining mathematical feature that distinguishes them from periodic crystals. The Loom and Weaving operate through the aperiodic but ordered tiling of space.

10. Astronomical Scale: Spherical Distributions in Space

The Thomson problem's solutions apply wherever objects are distributed on or near spherical surfaces:

Globular clusters: The approximately spherical distributions of stars in globular clusters represent gravitational energy minimisation — the gravitational analogue of electrostatic energy minimisation. While the 1/r potential differs from Coulomb's 1/r, the optimisation principle is identical.

Cosmic microwave background: The CMB represents the earliest observable distribution of energy on a spherical surface (the last-scattering surface). The angular power spectrum of CMB anisotropies — the pattern of hot and cold spots — has been analysed for geometric signatures consistent with icosahedral or dodecahedral symmetry.

The Poincaré dodecahedral space hypothesis: In 2003, Luminet et al. proposed that the topology of the universe might be a Poincaré dodecahedral space — a positively curved space whose fundamental domain is a regular dodecahedron (the dual of the icosahedron). This would explain the suppression of large-angle correlations in the CMB power spectrum. The dodecahedron's 12 pentagonal faces encode φ in every dimension.

Framework interpretation: If the fundamental domain of the universe is dodecahedral, then the golden ratio is encoded in the topology of space itself. The Thomson problem's preference for icosahedral geometry at the level of point charges on a sphere would be a local expression of a global geometric principle — the universe, at its deepest level, is structured by φ.

PART III: THE MATHEMATICAL BRIDGE

11. Why Phi Emerges from Energy Minimisation

The appearance of φ in Thomson solutions is not accidental. It follows from three mathematical properties:

Property 1: Phi solves the packing problem on curved surfaces.

On a flat surface, the most efficient packing is hexagonal (6-fold symmetry). On a curved surface, hexagonal packing cannot close — you need pentagonal defects. The transition from 6-fold to 5-fold coordination introduces φ because the pentagon's diagonal-to-side ratio is exactly φ. Every time nature accommodates curvature in a triangular lattice, it introduces φ.

Property 2: Phi is the most irrational number.

The golden ratio has the slowest-converging continued fraction expansion: [1; 1, 1, 1, ...]. This means that φ-based spacings avoid the near-coincidences that produce inefficient clustering. The golden angle ($360°/φ^2 \approx 137.508°$) produces the most uniform possible angular distribution — proven mathematically, not conjectured. This is why phyllotaxis uses it, why Thomson solutions for large N approximate it, and why NASA uses it for satellite mirror arrays.

Property 3: Phi connects the icosahedron to the dodecahedron through duality.

The icosahedron (12 vertices, 20 faces) and dodecahedron (20 vertices, 12 faces) are dual polyhedra — each is constructed by connecting the face centres of the other. Both are saturated with φ. The Thomson problem selects the icosahedron for energy minimisation; its dual, the dodecahedron, governs the topology of quasicrystals and potentially the universe itself. Together they form a φ-defined geometric pair that bridges scales.

12. The Dual-Algorithm Signature in Thomson Geometry

The framework identifies the Weaving (Fibonacci) and the Loom (Lucas) as complementary algorithms. The Thomson problem's geometry encodes both:

Fibonacci in the icosahedron:

- The icosahedron has 12 vertices, 30 edges, 20 faces
- $12 = F(3)^2 \times F(4) = 4 \times 3$
- $30 = F(5) \times F(3) \times F(4) = 5 \times 2 \times 3$
- $20 = F(4) \times F(3) \times F(5)/F(3) = \ldots$ or simply: $20 = 4 \times 5 = L(3) \times F(5)$
- Vertex coordinates: $(0, \pm 1, \pm\varphi)$ — where $\varphi = \lim(F(n+1)/F(n))$

Lucas in the dodecahedron (dual):

- The dodecahedron has 20 vertices, 30 edges, 12 faces
- Vertex coordinates include $(\pm 1, \pm 1, \pm 1)$ and $(0, \pm 1/\varphi, \pm\varphi)$ and $(\pm 1/\varphi, \pm\varphi, 0)$ and $(\pm\varphi, 0, \pm 1/\varphi)$
- The dihedral angle: $\arctan(2) \approx 116.565°$ — and $2 = L(1)$

The 12 pentagonal defects:

- Always 12, required by Euler's theorem ($V - E + F = 2$)
- 12 = the number that connects Fibonacci and Lucas: it is both 3×4 ($= F(4) \times L(3)$) and the product of the first structural outputs of both algorithms

The CK triangulation numbers:

- T = 1, 3, 4, 7, 12, 13...
- T = 1: F(1) or F(2)
- T = 3: F(4)
- T = 4: L(3)
- T = 7: L(4)
- T = 12: $F(3)^2 \times F(4)$
- T = 13: F(7)

The first six CK triangulation numbers are 1, 3, 4, 7, 12, 13. Of these: 1, 3, 13 are Fibonacci numbers; 1, 3, 4, 7 are Lucas numbers. **Five of the first six T-numbers are in the Fibonacci or Lucas sequence.** The sixth (12) is the product of consecutive terms from each.

13. The Base-60 Connection

The Thomson problem operates on a sphere — 360° of angular space. The framework identifies Base-60 as Loom output (Lucas structural encoding), and $360 = 6 \times 60 = 6 \times L(1) \times L(4) \times F(5)$.

The golden angle on a 360° sphere: $360°/\varphi^2 \approx 137.508°$

The complement of the golden angle: $360° - 222.492° = 137.508°$, or equivalently: $360°/\varphi = 222.492°$ and $360° - 360°/\varphi = 360°(1 - 1/\varphi) = 360°/\varphi^2 = 137.508°$

This means the golden angle is the Base-60 circle (360°) divided by φ^2 — the intersection of Loom (Base-60) and Weaving (φ). The two algorithms meet on the surface of a sphere.

The 60-fold symmetry in virus capsids directly reflects this: T = 1 capsids contain exactly 60 protein subunits, arranged with icosahedral symmetry. 60 = the Loom's structural number. The icosahedral geometry = the Weaving's optimisation output. Structure (60) × Geometry (φ) = virus capsid.

PART IV: THE FRAMEWORK SYNTHESIS

14. What the Thomson Problem Proves

The Thomson problem demonstrates something that no other single mathematical result does: **the golden ratio is not a coincidence, a curiosity, or an aesthetic preference. It is the inevitable output of energy minimisation on closed surfaces.**

This has profound implications for the framework:

Implication 1: The Weaving is not imposed — it is derived.

Fibonacci ratios in planetary orbits, atomic spectra, and molecular geometries are not arbitrary signatures "programmed" into nature. They are the mathematical consequence of systems minimising energy under spherical or toroidal constraints. The algorithm is energy minimisation itself.

Implication 2: The bridge across scales is real.

The same Thomson principle — identical point charges on a sphere seeking minimum energy — produces:

- Icosahedral electron shells in atoms (10^{-10} m)
- Icosahedral virus capsids (10^{-8} m)
- Icosahedral fullerene cages (10^{-9} m)
- Icosahedral quasicrystals (10^{-3} m and above)
- Potentially icosahedral/dodecahedral cosmic topology (10^{26} m)

That is a span of **36 orders of magnitude** — from sub-atomic to cosmological — unified by a single geometric principle rooted in the golden ratio.

Implication 3: The Loom and Weaving interact through topology.

Euler's theorem guarantees 12 pentagonal defects on any triangulated sphere. This is a topological invariant — it cannot be changed by continuous deformation. The number 12 (= F(4) × L(3)) is thus hardwired into the topology of closed surfaces. The interaction between Fibonacci and Lucas is not just algebraic — it is topological.

Implication 4: The mechanism is electromagnetic.

The Thomson problem is explicitly about electrostatic energy minimisation. The framework proposes that reality is a unified consciousness-EM field. The Thomson problem shows that electromagnetic optimisation spontaneously generates the framework's signature geometry. The medium (EM field) and the pattern (Fibonacci/Lucas/φ) are connected through the deepest available physical principle: minimum energy.

15. Testable Predictions

The Thomson bridge generates specific predictions:

1. **Any future discovery of a naturally occurring spherical energy-minimising system will exhibit icosahedral symmetry or golden-angle spacing.** This includes: engineered protein cages, self-assembled colloidal crystals, confined plasma structures, and gravitationally bound spherical systems.
2. **The magic numbers of the Thomson problem (12, 32, 72, 122, 132, 192...) will correspond to stability maxima in any confined spherical charge system.** This has already been confirmed for metallic clusters, quantum dots, and helium nanodroplets.
3. **Quasicrystalline order with icosahedral symmetry can emerge from single-component systems with short-range interactions.** This was confirmed in 2020 by molecular dynamics simulations — predicted by the principle that φ-geometry is the minimum-energy state, not a product of complicated multi-body forces.
4. **If the topology of the universe is measurably non-trivial, it will be dodecahedral (Poincaré space) or icosahedral — not cubic, octahedral, or any other symmetry.** The Thomson principle predicts that cosmic-scale energy minimisation favours the same φ-based geometry as atomic-scale minimisation.

Summary

The Thomson problem is the missing physical mechanism that explains why Fibonacci and Lucas numbers pervade nature. It demonstrates that:

1. **Energy minimisation on spherical surfaces spontaneously generates the golden ratio** through icosahedral geometry, pentagonal defects, and golden-angle spacing.
2. **This occurs at every scale of physical reality** — from atomic electron shells through virus capsids and fullerenes to quasicrystals and potentially cosmic topology — spanning 36+ orders of magnitude.
3. **The 12 pentagonal defects** required by Euler's theorem on any triangulated sphere provide a topological foundation for the framework's structural number 12 = F(4) × L(3).
4. **The golden angle** ($360°/\varphi^2$) represents the intersection of Base-60 (Loom) and the golden ratio (Weaving) on the surface of a sphere — the two algorithms meeting in angular space.
5. **The mechanism is electromagnetic** — the Thomson problem is explicitly about Coulomb energy minimisation, directly connecting the framework's EM field hypothesis to its Fibonacci/Lucas mathematical structure.
6. **The Caspar-Klug T-numbers** that govern virus architecture and fullerene stability are dominated by Fibonacci and Lucas numbers (1, 3, 4, 7, 12, 13...), providing an independent confirmation of the dual-algorithm signature.

The Thomson problem does not merely support the framework. It provides the physical mechanism — energy minimisation under geometric constraint — that makes the framework's predictions inevitable.

This document should be read alongside: The Galactic Signature (exoplanet resonance chains), The Harmony of Inevitability (Bayesian probability), Chemical Foundations (spectral analysis), and the Framework User Guide.

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